VALLIAMMAI ENGINEERING COLLEGE

SRM Nagar, Kattankulathur – 603 203.

DEPARTMENT OF MATHEMATICS

QUESTION BANK

SUBJECT : MA6151 – MATHEMATICS -1

SEM / YEAR:ISem/ I year B.E., B.Tech. (Common to all branches)

UNIT I - MATRICES

Eigenvalues and Eigenvectors of a real matrix - Characteristic equation - Properties of eigenvalues and eigenvectors - Statement and applications of Cayley-Hamilton Theorem - Diagonalization of matrices - Reduction of a quadratic form to canonical form by orthogonal transformation - Nature of quadratic forms.

Q.No.	Question	Bloom's Taxonomy Level	Domain
	PART – A		
1.	Find the sum and product of all the Eigen values of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	**BTL -2**	Understanding
2.	What are the Eigen values of the matrix A + 3I, if the Eigen values of the matrix $A = \begin{pmatrix} 1 & -2 \\ -5 & 4 \end{pmatrix}$ are 6 and -1? Why?	**BTL -1**	Remembering
3.	Find the Eigen vales of 3A + 2I, where $A = \begin{pmatrix} 5 & 4 \\ 0 & 2 \end{pmatrix}$	**BTL -2**	Understanding
4.	Find the Eigen values of the inverse of the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 4 \end{pmatrix}$	**BTL -2**	Understanding
5.	If λ is the Eigen value of the (a square) matrix A, then prove that λ^2 is the Eigen value of A^2	**BTL -3**	Applying
6.	If the Eigen values of the matrix A of order 3 x 3 are 2, 3 and 1, then find the Eigen values of adjoint of A.	**BTL -2**	Understanding
7.	Find the values of a and b such that the matrix $\begin{pmatrix} a & 4 \\ 1 & b \end{pmatrix}$ has 3 and -2 its Eigen values	**BTL -2**	Understanding
8.	The product of two eigenvalues of the matrix $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ is 16. Find the third eigenvalue of A.	**BTL -2**	Understanding
9.	If 3 and 6 are two Eigen values of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$, write down all the Eigen values of A^{-1}	**BTL -3**	Applying
10.	State Cayley – Hamilton theorem.	**BTL -1**	Remembering

11.	Find the Eigen values of $A = \begin{pmatrix} 3 & 1 & 2 \\ 0 & -1 & 4 \\ 0 & 0 & -5 \end{pmatrix}$. Also find the Eigen values of -3A	**BTL -2**	Understanding
12.	Find the symmetric matrix A, whose eigen values are 1 and 3 with corresponding eigenvectors $\begin{pmatrix} 1 \\ -1 \end{pmatrix} and \begin{pmatrix} 1 \\ 1 \end{pmatrix}$	**BTL -2**	Understanding
13.	Find the Eigen values of A^{-1} where $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$	**BTL -2**	Understanding
14.	If 2, -1, -3 are the Eigen values of the matrix A, then find the Eigen values of the matrix $A^2 - 2I$	**BTL -2**	Understanding
15.	Can $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be diagonalised? Why?	**BTL -4**	Analyzing
16.	What is the nature of the quadratic form $x^2 + y^2 + z^2$ in four variables?	**BTL -2**	Understanding
17.	Identify the nature, index and signature of the quadratic form $2x_1x_2 + 2x_2x_3 + 2x_3x_1$	**BTL -4**	Analyzing
18.	Give the nature of the quadratic form whose matrix is $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$	**BTL -4**	Analyzing
19.	Write down the matrix of the quadratic form $2x^2 + 8z^2 + 4xy + 10xz - 2yz$	**BTL -3**	Applying
20.	Write down the quadratic form corresponding to the matrix $A = \begin{pmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{pmatrix}$	**BTL -3**	Applying
	PART – B		
1.(a)	Find the Eigen values and Eigen vectors of $\begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$		Remembering
1. (b)	If A = $\begin{pmatrix} 7 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$, verify Cayley – Hamilton Theorem and hence find A^{-1}	**BTL -4**	Applying
2. (a)	Find the Eigen values and Eigen vectors of a matrix $A = \begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$	**BTL -2**	Understanding
2.(b)	Reduce the quadratic form $x^2+5y^2+z^2+2xy+2yz+6zx$ into canonical form and hence find its rank.	**BTL -3**	Applying
3. (a)	State Cayley – Hamilton theorem and using it, find the matrix represented by $A^8-5A^7+7A^6-3A^5+A^4-5A^3+8A^2-2A+I$ when $A = \begin{pmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{pmatrix}$	**BTL -2**	Understanding
3.(b)	Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ into canonical form by the orthogonal	**BTL -3**	Applying

	transformation		
4. (a)	If λ_i for (i = 1, 2, …, n) are the non-zero Eigen values of A , then prove that (1) $K\lambda_i$ are the Eigen Values of KA, where K being a non-zero scalar; (2) $\frac{1}{\lambda i}$are the Eigen values of A^{-1} , (3) ${\lambda_i}^K$ are the Eigen Values of A^K	**BTL -4**	Analyzing
4.(b)	Reduce the quadratic form $3x^2+ 5y^2+ 3z^2- 2yz + 2zx - 2xy$ into canonical form through orthogonal transformation.	**BTL -3**	Applying
5. (a)	Using Cayley-Hamilton theorem find A^{-1} and A^4, if $A = \begin{pmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{pmatrix}$	**BTL -1**	Remembering
5.(b)	Determine a diagonal matrix orthogonally similar to the real symmetric matrix$\begin{pmatrix} 3 & 1 & 1 \\ 1 & 3 & -1 \\ 1 & -1 & 3 \end{pmatrix}$	**BTL -2**	Understanding
6. (a)	Verify Cayley – Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$. Also compute A^{-1}	**BTL -4**	Analyzing
6.(b)	Reduce the quadratic form $2x_1x_2 + 2x_1x_3 - 2x_2x_3$ into the canonical form by an orthogonal reduction. Also find its nature	**BTL -3**	Applying
7. (a)	Diagonalize the matrix $A = \begin{pmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{pmatrix}$	**BTL -4**	Analyzing
7. (b)	Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & -2 & 3 \\ 2 & 4 & -2 \\ -1 & 1 & 2 \end{pmatrix}$	**BTL -4**	Analyzing
8. (a)	Show that the matrix$\begin{pmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 2 & 0 & 3 \end{pmatrix}$satisfies its own characteristic equation. Find also its inverse	**BTL -4**	Analyzing
8.(b)	Reduce the quadratic form $2x_1x_2 + 2x_2x_3 + 2x_3x_1$ into canonical form	**BTL -3**	Applying
9. (a)	The eigenvectors of a 3x3 real symmetric matrix A corresponding to the eigenvalues 2,3, 6 are $(1, 0, -1)^T$, $(1, 1, 1)^T$and $(-1,2,-1)^T$respectively. Find the matrix A.	**BTL -2**	Understanding
9.(b)	Show that the matrix$A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$satisfies its own characteristic equation. Find also its inverse.	**BTL -4**	Analyzing
10.(a)	If the eigen values of $A = \begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$are 0, 3, 15, find the eigenvectors	**BTL -2**	Understanding
10.(b)	Find A^n using Cayley-Hamilton theorem, taking$A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Hence find A^3	**BTL -2**	Understanding
11.(a)	Find the Eigen vales and Eigen vectors of the matrix	**BTL -2**	Understanding

	$\begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$		
11.(b)	Determine a diagonal matrix orthogonally similar to the real symmetric matrix $\begin{pmatrix} 3 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{pmatrix}$	**BTL -2**	Understanding
12.(a)	Using Cayley – Hamilton theorem find A^4 for the matrix $A = \begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$	**BTL -2**	Understanding
12.(b)	Determine a diagonal matrix orthogonally similar to the real symmetric matrix $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$	**BTL -2**	Understanding
13.(a)	Using Cayley Hamilton theorem find the inverse of $\begin{pmatrix} 5 & 3 \\ 1 & 3 \end{pmatrix}$	**BTL -2**	Understanding
13.(b)	Reduce the quadratic form $x^2+ y^2+ z^2- 2xy - 2yz - 2zx$ into canonical form through an orthogonal transformation. Write down the transformation	**BTL -3**	Applying
14.(a)	Using Cayley Hamilton theorem for the matrix $\begin{pmatrix} 1 & 4 \\ 2 & 3 \end{pmatrix}$ find the value of the polynomial $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$	**BTL -2**	Understanding
14.(b)	Reduce the Quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to canonical form by orthogonal reduction and states its nature	**BTL -3**	Applying

UNIT -IISEQUENCE AND SERIES

Sequences: Definition and examples – Series: Types and Convergence – Series of positive terms – Tests of convergence: Comparison test, Integral test and D'Alembert's ratio test – Alternating series – Leibnitz's test – Series of positive and negative terms – Absolute and conditional convergence.

Q.No.	Question	Bloom's Taxonomy Level	Domain
	PART - A		
1.	Distinguish between a sequence and series.	**BTL -1**	Remembering
2.	Discuss the convergence of the sequence $\{a_n\}$where $a_n = \frac{n+1}{n}$.	**BTL -1**	Remembering
3.	Discuss the convergence of the sequence $\{S_n\}$where $S_n = \frac{n^2-n}{2n^2+n}$.	**BTL -1**	Remembering
4.	Examine the convergence of the series $\sum_{n=1}^{\infty} \log \frac{n}{n+1}$.	**BTL -1**	Remembering
5.	Test the convergence of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \cdots$	**BTL -3**	Applying
6.	Using Comparison test, prove that the series $\frac{1}{1 \cdot 3} + \frac{2}{3 \cdot 5} + \frac{3}{5 \cdot 7} + \cdots$ is divergent.	**BTL -3**	Applying
7.	Find the nature of the series $1 + 2 + 3 + \cdots$.	**BTL -2**	Understanding
8.	Test the convergence of the series $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \cdots + \frac{1}{\sqrt{n}} + \ldots$	**BTL -3**	Applying
9.	Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2n^3}{n^4+1}$.	**BTL -4**	Analyzing
10.	Test the convergence of the series $\frac{1^2 \cdot 2^2}{1!} + \frac{2^2 \cdot 3^2}{2!} + \frac{3^2 \cdot 4^2}{3!} + \cdots$	**BTL -3**	Applying

11.	Define integral test.	**BTL -1**	Remembering
12.	Using integral test determine the convergence of the series $1+\frac{1}{3}+\frac{1}{5}+\cdots+\frac{1}{2n-1}+\cdots$.	**BTL -2**	Understanding
13.	Test the convergence of the series $1+\frac{1}{2^2}-\frac{1}{3^2}-\frac{1}{4^2}+\frac{1}{5^2}+\frac{1}{6^2}-\frac{1}{7^2}-\frac{1}{8^2}+\cdots$	**BTL -3**	Applying
14.	Examine the convergence of the series $1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+\cdots$	**BTL -2**	Understanding
15.	Test the convergence of the series $sin\pi+\frac{1}{4}sin\frac{\pi}{2}+\frac{1}{9}sin\frac{\pi}{3}+\cdots$	**BTL -3**	Applying
16.	Test the convergence of the series $\sum_{n=0}^{\infty}e^{-n^2}$	**BTL -4**	Analyzing
17.	Test the convergence of the series $\sum_{n=1}^{\infty}n\ e^{-n^2}$	**BTL -4**	Analyzing
18.	Test the convergence of the series $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{\sqrt{n}}$.	**BTL -4**	Analyzing
19.	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$.	**BTL -4**	Analyzing
20.	Give an example for conditionally convergent series.	**BTL -6**	Creating
	PART -B		
1(a)	Show by direct summation of n terms that the series $\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots$ is convergent		
1. (b)	Test the convergence of the series $1+\frac{3}{7}x+\frac{3\cdot 6}{7\cdot 10}x^2+\frac{3\cdot 6\cdot 9}{7\cdot 10\cdot 13}x^3+\cdots$	**BTL -3**	Analyzing
2. (a)	Using Comparison test, examine the convergence or divergence of $\frac{1}{1\cdot 2\cdot 3}+\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}+\cdots$	**BTL -2**	Understanding
2.(b)	Examine the convergence and the divergence of the series $1+\frac{2}{5}x+\frac{6}{9}x^2+\frac{14}{17}x^3+\cdots+\frac{2^n-2}{2^n+1}x^{n-1}+\cdots x>0.$	**BTL -2**	Understanding
3.(a)	Test the convergence of the series $\sum_{n=1}^{\infty}(\ \sqrt{n^4+1}-n^2\ )$	**BTL -4**	Analyzing
3.(b)	Discuss the convergence of the series $\sum\frac{\sqrt{n}}{\sqrt{n^2+1}}x^n$	**BTL -2**	Understanding
4. (a)	Test the convergence of the series $\frac{1}{4\cdot 7\cdot 10}+\frac{4}{7\cdot 10\cdot 13}+\frac{9}{10\cdot 13\cdot 16}+\cdots$	**BTL -4**	Applying
4.(b)	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n}{n^2+1}x^n,\ x>0$	**BTL -3**	Analyzing
5. (a)	Examine convergence of the series $\sum_{n=1}^{\infty}(\ \sqrt[3]{n^3+1}-n\ )$	**BTL -4**	Applying
5.(b)	Test the convergence of the series $1+\frac{2^p}{2!}+\frac{3^p}{3!}+\frac{4^p}{4!}+\cdots$ by D'Alembert's ratio test	**BTL -3**	Analyzing
6. (a)	Examine the convergence of the series $\frac{1}{1+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\cdots$	**BTL -3**	Analyzing
6.(b)	Test the convergence of the series $\frac{4}{18}+\frac{4\ .\ 12}{18\ .\ 27}+\frac{4\ .\ 12\ .\ 20}{18\ .\ 27\ .\ 36}+\cdots$by Ratio test	**BTL -3**	Analyzing
7. (a)	Using D' Alembert ratio test, examine the convergence or divergence of $x+2x^2+3x^3+\cdots$	**BTL -3**	Analyzing
7. (b)	Test for absolute convergence of the series $1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots$	**BTL -2**	Understanding
8. (a)	Prove that the harmonic series is divergent	**BTL -1**	Remembering

8.(b)	Prove that the series $\frac{1}{1\ 3}+\frac{2}{3\ .\ 5}+\frac{3}{5\ .\ 7}+\frac{4}{7\ .\ 9}+\cdots$ is divergent by Ratio test	**BTL -5**	Evaluating
9. (a)	Test the convergence of the series $\sum_{n=1}^{\infty}\frac{n}{(n^2+1)^2}$ by Integral test	**BTL -3**	Applying
9.(b)	Determine convergence of an alternating series $\sum_{n=1}^{\infty}\frac{\cos n\pi}{n^2+1}$and also test for absolute and conditional convergence.	**BTL -3**	Applying
10.(a)	Test the convergence of the series $\sum_{n=3}^{\infty}\frac{1}{n\sqrt{\log n}}$	**BTL -4**	Applying
10.(b)	Find integral of the convergence $\frac{x}{1+x}-\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}-\frac{x^4}{1+x^4}\cdots$ $(0<x<1)$	**BTL -2**	Understanding
11.(a)	Test the convergence of the series $\sum_{n=2}^{\infty}\frac{1}{n\log n}$	**BTL -4**	Applying
11.(b)	Find the convergence of the series $x-\frac{x^2}{\sqrt{2}}+\frac{x^3}{\sqrt{3}}-\frac{x^4}{\sqrt{4}}+\cdots\infty$	**BTL -2**	Understanding
12.(a)	Find the nature of the series $\sum_{n=2}^{\infty}\frac{1}{n(\log n)^p}$ by Cauchy's integral test.	**BTL -5**	Evaluating
12.(b)	Test the convergence and divergence of the series $\frac{1}{\sqrt{2}+1}-\frac{1}{\sqrt{3}+1}+\frac{1}{\sqrt{4}+1}-\frac{1}{\sqrt{5}+1}+\cdots$	**BTL -3**	Analyzing
13.(a)	Test the convergence of the series $\sum_{n=1}^{\infty}n^2e^{-n^3}$	**BTL -3**	Analyzing
13.(b)	Test the convergence or divergence of $\frac{1}{1\cdot 2}-\frac{1}{3\cdot 4}+\frac{1}{5\cdot 6}-\frac{1}{7\cdot 8}+\cdots$	**BTL -4**	Applying
14.(a)	Discuss the convergence of $\sum_{n=1}^{\infty}\frac{1}{n^2}\sin\left(\frac{1}{n}\right)$	**BTL -3**	Analyzing
14.(b)	Test the convergence of the series $\frac{1}{2^3}-\frac{1}{3^3}(1+2)+\frac{1}{4^3}(1+2+3)-\frac{1}{5^3}(1+2+3+4)+\cdots$	**BTL -4**	Applying

UNIT – III APPLICATION OF DIFFERENTIAL CALCULUS

Curvature in Cartesian co-ordinates – Centre and radius of curvature – Circle of curvature – Evolutes – Envelopes - Evolute as envelope of normals.

Q.No.	Question	Bloom's Taxonomy Level	Domain
	PART - A		
1.	What is Circle of Curvature?	**BTL -1**	Remembering
2.	Find the curvature of the curve of the curve $2x^2+2y^2+5x-2y+1=0.$	**BTL -2**	Understanding
3.	Find the radius of curvature of the curve $xy=C^2$ at (c, c).	**BTL -2**	Understanding
4.	Find radius of curvature of curve $x^2+y^2-4x+2y-8=0.$	**BTL -2**	Understanding
5.	What is the radius of curvature of the curve $x^4+y^4=2$ at the point (1,1)	**BTL -2**	Understanding
6.	Find the radius of curvature for $y=e^x$ at the point where it is cuts the y-axis.	**BTL -2**	Understanding
7.	Find the curvature of the curve $x^2+y^2+2x+2y-1=0$ at (x,y).	**BTL -2**	Understanding
8.	What is the radius of curvature of the curve $x^2+y^2=25$ at the point (4,3).	**BTL -3**	Analyzing
9.	Define curvature of a Plane curve.	**BTL -1**	Remembering

10.	What is the curvature of the circle $(x-1)^2+(y+2)^2=16$ at any point on it .	**BTL -4**	Applying
11.	Find the envelope of $x\cos\theta + y\sin\theta = a$, where θ is a parameter.	**BTL -2**	Understanding
12.	Find the envelope of the family of circles $(x-\alpha)^2+y^2=4\alpha$, where α is the parameter	**BTL -2**	Understanding
13.	Find the envelope of the lines $\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta=1$, θ being the parameter.	**BTL -2**	Understanding
14.	Find the envelope of the lines $x\,\mathrm{cosec}\,\theta - y\cot\theta = a$, θ being the parameter.	**BTL -2**	Understanding
15.	Find the envelope of the family of lines $\frac{x}{t}+yt=2c$, t being a parameter.	**BTL -2**	Understanding
16.	Write the properties of evolutes.	**BTL -1**	Remembering
17.	Find the envelope of $x - y\sin\theta = a\cos\theta$, where θ is a parameter.	**BTL -2**	Understanding
18.	Find the envelope of the family of straight lines $y=mx+\frac{a}{m}$. where a is a parameter.	**BTL -2**	Understanding
19.	Find the envelope of the family of straight lines $y = mx - 2am - am^3$ where m is a parameter.	**BTL -2**	Understanding
20.	Find the Envelope of $y=mx\pm\sqrt{a^2m^2+b^2}$. where m is a parameter.	**BTL -2**	Understanding
	PART -B		
1.(a)	Find radius of the curvature of $x^{2/3}+y^{2/3}=a^{2/3}$.	**BTL -2**	Understanding
1. (b)	Find the centre of curvature of $x^3+y^3=6xy$ at (3, 3).	**BTL -2**	Understanding
2. (a)	Show that the radius of curvature at any point of thecatenary $y=c\cosh\left(\frac{x}{c}\right)$ is C. Also find ρ at (0, c).	**BTL -4**	Applying
2.(b)	Prove that for the curve $y=\frac{ax}{a+x}$, $\left(\frac{2\rho}{a}\right)^{\frac{2}{3}}=\left(\frac{x}{y}\right)^2+\left(\frac{y}{x}\right)^2$.	**BTL -4**	Applying
3. (a)	Find the radius of curvature at any point of the Cycloid $x=a(\theta+\sin\theta),\ y=a(1-\cos\theta)$.	**BTL -2**	Understanding
3.(b)	Find the circle of curvature at $\left(\frac{a}{4},\frac{a}{4}\right)$ on $\sqrt{x}+\sqrt{y}=\sqrt{a}$.	**BTL -2**	Understanding
4. (a)	Find the radius of curvature and centre of curvature of the Parabola $y^2=4ax$. at the point t.	**BTL -2**	Understanding
4.(b)	If the centre of curvature of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, at one end on the minor axis lie at the other end, Prove that the eccentricity of	**BTL -4**	Applying

	the ellipse is $\frac{1}{\sqrt{2}}$.		
5. (a)	Find the radius of curvature of the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy.$	**BTL -2**	Understanding
5.(b)	Find the equation of the circle of curvature of the rectangular hyperbola $xy = 12$ at (3, 4).	**BTL -2**	Understanding
6. (a)	Obtain the evoluteof $x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta).$	**BTL -6**	Creating
6.(b)	Obtain the envelope $\frac{x}{a} + \frac{y}{b} = 1,$ if $(i) a^2 + b^2 = c^2, (ii) ab = c^2.$	**BTL -6**	Creating
7. (a)	Find the equation of the evolute of the parabola $x^2 = 4ay,$ treating it as the envelope of normal.	**BTL -2**	Understanding
7. (b)	Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ where a and b are connected by the relation $a^n + b^n = c^n$, c being a constant.	**BTL -2**	Understanding
8. (a)	Considering the evolute as the envelope of normal, Find the evolute of the tractrix $x = a\left(\cos\theta + \log\tan\frac{\theta}{2}\right),\ y = a\sin\theta.$	**BTL -2**	Understanding
8.(b)	Prove that the radius of curvature at any point (x, y) on $\left(\frac{x}{a}\right)^{\frac{1}{2}} + \left(\frac{y}{b}\right)^{\frac{1}{2}} = 1$ *is* $\rho = \frac{2(ax+by)^{\frac{3}{2}}}{ab}.$	**BTL -4**	Applying
9. (a)	Show that the radius of curvature at any point of the curve $x = ae^{\theta}(\sin\theta - \cos\theta),\ y = ae^{\theta}(\sin\theta + \cos\theta)$ is twice the perpendicular distance from the origin to the tangent at the point.	**BTL -4**	Applying
9.(b)	Find the equation of the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$	**BTL -2**	Understanding
10.(a)	Find the evolute of the rectangular hyperbola $xy = C^2.$	**BTL -2**	Understanding
10.(b)	Find the points on the parabola $y^2 = 4x.$ at which the radius of curvature is $4\sqrt{2}.$	**BTL -2**	Understanding
11.(a)	Find the equation of the evolute of the parabola $y^2 = 4ax.$	**BTL -2**	Understanding
11.(b)	Considering the evolute as the envelope of the normal, find the evolute of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}.$	**BTL -2**	Understanding
12.(a)	Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$ where a and b are connected by the relation $a^2 + b^2 = c^2$, c being a constant.	**BTL -2**	Understanding
12.(b)	Find the equation of the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$	**BTL -2**	Understanding
13.(a)	Find the envelope of $\frac{x}{l} + \frac{y}{m} = 1,$ where l & m are connected by	**BTL -2**	Understanding

	the relation $\frac{l}{a}+\frac{m}{b}=1,$ where a and b are constants.		
13.(b)	Show that the evolute of the cycloid $x=a(\theta-\sin\theta), y=a(1-\cos\theta)$ is another cycloid.	**BTL -4**	Applying
14.(a)	Find the envelope of the family of straight lines $x\cos\alpha+y\sin\alpha=c\sin\alpha,\cos\alpha,$ α being the parameter.	**BTL -2**	Understanding
14.(b)	Find the envelope of the straight line $\frac{x}{a}+\frac{y}{b}=1,$ where the parameter a and b are connected by the relation $a^n+b^n=c^n,$ c being a constant.	**BTL -2**	Understanding

UNIT – IV DIFFERENTIAL CALCULUS OF SEVERAL VARIABLES

Limits and Continuity – Partial derivatives – Total derivative – Differentiation of implicit functions – Jacobian and properties – Taylor's series for functions of two variables – Maxima and minima of functions of two variables – Lagrange's method of undetermined multipliers

Q.No.	Question	Bloom's Taxonomy Level	Domain
	PART - A		
1.	Evaluate $\lim_{\substack{x\to\infty\\ y\to 2}}\frac{xy+5}{x^2+2y^2}.$	**BTL -5**	Evaluating
2.	If $u=\frac{y}{z}+\frac{z}{x}+\frac{x}{y}$, *then find* $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}.$	**BTL -2**	Understanding
3.	If $u=f(x-y, y-z, z-x)$, *then find* $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}.$	**BTL -2**	Understanding
4.	If $x^y+y^x=1$, then find $\frac{dy}{dx}$.	**BTL -2**	Understanding
5.	Find the value of $\frac{du}{dt}$, given $u=y^2-4ax,\ x=at^2.$	**BTL -2**	Understanding
6.	If $u=x^3y^2+x^2y^3$ *where* $x=at^2$ *and* $y=2at$, *then find* $\frac{du}{dt}.$	**BTL -2**	Understanding
7.	Find $\frac{du}{dt}$ *if* $u=\sin\left(\frac{x}{y}\right)$, *where* $x=e^t, y=t^2.$	**BTL -2**	Understanding
8.	Find $\frac{du}{dt}$ *if* $u=\frac{x}{y},$ *where* $x=e^t, y=\log t.$	**BTL -2**	Understanding
9.	Find the Jacobian $\frac{\partial(r,\theta)}{\partial(x,y)}$, *if* $x=r\cos\theta$ & $y=r\sin\theta.$	**BTL -2**	Understanding
10.	Find the Jacobian $\frac{\partial(u,v)}{\partial(r,\theta)}$, *if* $x=r\cos\theta$ & $y=r\sin\theta, u=2xy, v=x^2-y^2$, without actual substitution.	**BTL -2**	Understanding
11.	If $u=\frac{y^2}{2x}$ *and* $v=\frac{x^2+y^2}{2x},$ *find* $\frac{\partial(u,v)}{\partial(x,y)}.$	**BTL -2**	Understanding

12.	If $x = u(1+v), y = v(1+u)$. *Find* $\frac{\partial(x,y)}{\partial(u,v)}$.	**BTL -2**	Understanding
13.	If $u = x^y$ show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$	**BTL -4**	Applying
14.	If u = $\frac{x+y}{1-xy}$ *and* $v = \tan^{-1} x + \tan^{-1} y$, *find* $\frac{\partial(u,v)}{\partial(x,y)}$	**BTL -4**	Applying
15.	Find the Taylor series expansion of x^y near the point $(1,1)$ up to first term	**BTL -4**	Applying
16.	Expand $xy + 2x - 3y + 2$ *in powers of* $(x-1)\&(y+2)$, using Taylor's theorem up to first degree form	**BTL -3**	Analyzing
17.	Find the Stationary points of $f(x,y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x.$	**BTL -2**	Understanding
18.	Find the Stationary points of $x^2 - xy + y^2 - 2x + y.$	**BTL -2**	Understanding
19.	State the Sufficient condition for $f(x,y)$ to be extremum at a point	**BTL -1**	Remembering
20.	Find the minimum point of f(x,y) = $x^2 + y^2 + 6x + 12$.	**BTL -2**	Understanding
	PART -B		
1.(a)	If u = log ($x^2 + y^2$) + $\tan^{-1}\left(\frac{y}{x}\right)$, prove that $u_{xx} + u_{yy} = 0$	**BTL -4**	Applying
1. (b)	If $u = \frac{yz}{x}, v = \frac{zx}{y}$ *and* $w = \frac{xy}{z}$, *find* $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.	**BTL -2**	Understanding
2. (a)	If $u = \tan^{-1}\frac{y}{x} - y^2 \tan^{-1}\frac{x}{y}$, then prove that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$	**BTL -4**	Applying
2.(b)	Find the Jacobian of $\frac{\partial(x,y,z)}{\partial(r,\theta,\varphi)}$ of the transformation $x = r\sin\theta\cos\varphi, y = r\sin\theta\sin\varphi, z = r\cos\theta$	**BTL -2**	Understanding
3. (a)	If z is a function of x and y and $x = e^u + e^{-v}, y = e^{-u} - e^v$ then show that $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v} = x\frac{\partial z}{\partial x} - y\frac{\partial z}{\partial y}$	**BTL -4**	Applying
3.(b)	Find the shortest & longest distance from the point(1, 2,-1) to the sphere $x^2 + y^2 + z^2 = 24$. Using Lagrange's multiplier method of constrained Maxima and Minima	**BTL -2**	Understanding
4. (a)	*If* $x + y + z = u, y + z = uv, z = uvw,$ *prove that* $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$	**BTL -4**	Applying
4.(b)	*If* $u = f(x,y)$ *where* $x = r\cos\theta, y = r\sin\theta$ *prove that* $\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2 = \left(\frac{\partial u}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial u}{\partial \theta}\right)^2$	**BTL -3**	Analyzing
5. (a)	Transform equation $z_{xx} + 2z_{xy} + z_{yy} = 0$ by changing the independent variables using $u = x - y$ *and* $v = x + y.$	**BTL -3**	Analyzing
5.(b)	Verify whether the following functions are functionally dependent, and if so, find the relation between them $u = \frac{x+y}{1-xy}, v = tan^{-1}x + tan^{-1}y$	**BTL -4**	Applying

6. (a)	*If* $z = f(x, y)$ *where* $x = u^2 - v^2$ $y = 2uv$ *provethat* $\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = 4(u^2 + v^2)\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$	**BTL -4**	Applying
6.(b)	Expand $\sin xy$ *at* $\left(1, \frac{\pi}{2}\right)$ upto second degree terms using Taylor's series.	**BTL -3**	Analyzing
7. (a)	Expand $e^x \log(1+y)$ in powers of x & y up to terms of third degree terms using Taylor's series	**BTL -3**	Analyzing
7. (b)	Discuss the maxima and minima of $f(x,y) = x^3y^2(1 - x - y)$.	**BTL -1**	Remembering
8. (a)	Find the Taylors series expansion of $e^x \, siny$ at the point (-1, $\frac{\pi}{4}$) up to the third degree terms	**BTL -2**	Understanding
8.(b)	Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$.	**BTL -2**	Understanding
9. (a)	Expand $\tan^{-1}\frac{y}{x}$ in the neighborhood of (1, 1)	**BTL -3**	Analyzing
9.(b)	Find the Maximum value of $x^m y^n z^p$ *when* $x + y + z = a$.	**BTL -2**	Understanding
10.(a)	Find the Taylors series expansion of $x^2y^2+2x^2y+3xy^2$ in powers of $(x+2)$ and $(y-1)$ upto 3rd degree	**BTL -2**	Understanding
10.(b)	Find the maximum value of $\sin x \sin y \sin(x+y)$ *where* $0 < x, y < \pi$.	**BTL -2**	Understanding
11.(a)	Expand e^{xy} in powers of $(x-1)$ and $(y-1)$ upto third degree terms by Taylor's series	**BTL -3**	Analyzing
11.(b)	Find the minimum value of x^2yz^3 *subject to* $2x + y + 3z = a$.	**BTL -2**	Understanding
12.(a)	Expand Taylor's series of $x^3 + y^3 + xy^2$ in powers of $(x-1)$ and $(y-2)$	**BTL -3**	Analyzing
12.(b)	Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$.	**BTL -2**	Understanding
13.(a)	Find the extremum value of $f(x,y) = x^3 + y^3 - 3x - 12y + 20$.	**BTL -2**	Understanding
13.(b)	If $u = xy + yz + zx, v = x^2 + y^2 + z^2$ *and* $w = x + y + z$, determine whether there is a functional relationship between u, v, w and if so, find it.	**BTL -5**	Evaluating
14.(a)	A rectangular box open at the top is to have volume of 32 cm. Find the dimension of the box requiring least material for its Construction	**BTL -6**	Creating
14.(b)	If $u = xyz, v = x^2 + y^2 + z^2$ *and* $w = x + y + z$ then find $\frac{\partial(x,y,z)}{\partial(u,v,w)}$.	**BTL -2**	Understanding

UNIT – VMULTIPLE INTEGRALS

Double integrals in Cartesian and polar coordinates – Change of order of integration – Area enclosed by plane curves – Change of variables in double integrals – Area of a curved surface - Triple integrals – Volume of Solids

Q.No.	Question	Bloom's Taxonomy Level	Domain
	PART - A		
1.	Evaluate $\int_2^3 \int_1^2 \frac{dxdy}{xy}$	**BTL -5**	Evaluating
2.	Evaluate $\int_0^{\pi/2} \int_0^{sin\theta} rdrd\theta$	**BTL -5**	Evaluating
3.	Find the area bounded by the lines $x = 0, y = 1\ and y = x$	**BTL -2**	Understanding
4.	Evaluate $\int_0^{\pi} \int_0^{a} rdrd\theta$	**BTL -5**	Evaluating
5.	Evaluate $\int_0^5 \int_0^2 (x^2 + y^2)dxdy$	**BTL -5**	Evaluating
6.	Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} dydx$	**BTL -5**	Evaluating
7.	Evaluate $\int_0^1 \int_1^2 xydxdy$	**BTL -5**	Evaluating
8.	Evaluate $\int_0^{\pi} \int_0^5 r^4 sin\theta drd\theta$	**BTL -5**	Evaluating
9.	Evaluate $\int_0^2 \int_0^x \frac{dxdy}{x^2+y^2}$	**BTL -5**	Evaluating
10.	Evaluate $\int\int dxdy$ over the region bounded by $x = 0, x = 2$, $y = 0\ and y = 2$	**BTL -5**	Evaluating
11.	Change the order of integration $\int_0^1 \int_0^y f(x, y)\ dxdy$	**BTL -4**	Applying
12.	Change the order of integration $\int_0^1 \int_0^x f(x, y)\ dxdy$	**BTL -4**	Applying
13.	Change the order of integration $\int_0^{\infty} \int_x^{\infty} f(x, y)\ dxdy$	**BTL -4**	Applying
14.	Evaluate $\int\int\int (x + y + z)\ dxdydz$ over the region bounded by $x = 0, x = 1$, $y = 0\ and y = 1, z = 0, z = 1$	**BTL -5**	Evaluating
15.	Write down the double integral to find the area of the circles $r = 2sin\theta, r = 4sin\theta$	**BTL -1**	Remembering
16.	Evaluate $\int_0^1 \int_x^{\sqrt{x}} xy\ (x + y)dydx$	**BTL -5**	Evaluating
17.	Evaluate $\int_0^1 \int_0^{x^2} (x^2 + y^2)dydx$	**BTL -5**	Evaluating
18.	Evaluate $\int_0^3 \int_0^2 \int_0^1 \int (x + y + z)\ dzdydx$	**BTL -5**	Evaluating
19.	Evaluate $\int_0^1 dx \int_0^2 dy \int_0^3 xyzdz$	**BTL -5**	Evaluating
20.	Evaluate $\int_a^b \int_c^d \int_f^g e^{x+y+z} dzdydx$	**BTL -5**	Evaluating
	PART-B		
1.(a)	Evaluate $\iint xydxdy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$	**BTL -5**	Evaluating
1. (b)	Change the order of integration $\int_0^{\infty} \int_0^y ye^{\frac{-y^2}{x}} dxdy$ and hence evaluate it	**BTL -4**	Applying

2. (a)	Evaluate $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}}(x^2+y^2)dydx$	**BTL -5**	Evaluating
2.(b)	By change the order of integration and evaluate $\int_0^2\int_{x^2}^{2-x} xy\,dydx$	**BTL -4**	Applying
3. (a)	Evaluate $\int\int xy\,dxdy$ over the positive quadrant of the circle $x^2+y^2=a^2$	**BTL -5**	Evaluating
3.(b)	Change the order of integration $\int_0^a\int_{\frac{x^2}{a}}^{2a-x} xydxdy$ and hence evaluate it	**BTL -4**	Applying
4. (a)	By changing in to polar Co – ordinates , evaluate $\int_0^{\infty}\int_0^{\infty} e^{-(x^2+y^2)}\,dxdy$	**BTL -4**	Applying
4.(b)	Change the order of integration $\int_0^1\int_{x^2}^{2-x} xydxdy$ and hence evaluate it	**BTL -4**	Applying
5. (a)	Evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}}(x^2+y^2)dydx$ by changing into polar co – ordinates	**BTL -5**	Evaluating
5.(b)	Change the order of integration $\int_0^a\int_y^a \frac{x}{\sqrt{x^2+y^2}}dydx$ and hence evaluate it	**BTL -4**	Applying
6. (a)	Change in to polar Co – ordinates and evaluate $\int_0^a\int_y^a \frac{x}{\sqrt{x^2+y^2}}dxdy$	**BTL -4**	Applying
6.(b)	Find the area of the cardioids $r=a(1+\cos\theta)$	**BTL -2**	Understanding
7. (a)	Change in to polar Co – ordinates and evaluate $\int_0^2\int_0^{\sqrt{2x-x^2}} \frac{x}{\sqrt{x^2+y^2}}dydx$	**BTL -4**	Applying
7. (b)	Find the volume of the tetrahedron bounded by the coordinate planes and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.	**BTL -2**	Understanding
8. (a)	Find the area enclosed by the curve $y^2=4ax$ and the lines $x+y=3a, y=0$.	**BTL -2**	Understanding
8.(b)	Evaluate $\iiint_V \frac{dzdydx}{(x+y+z+1)^3}$ where V is the region bounded by $x=0, y=0, z=0$and $x+y+z=1$.	**BTL -5**	Evaluating
9. (a)	Find the area included between the curves $y^2=4x$ and $x^2=4y$	**BTL -2**	Understanding
9.(b)	Evaluate $\int_1^e\int_1^{logy}\int_1^{e^x} logzdzdydx$	**BTL -5**	Evaluating
10.(a)	Find the smaller area bounded by $y=2$and $x^2+y^2=4$.	**BTL -2**	Understanding
10.(b)	Find the volume of the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$	**BTL -2**	Understanding
11.(a)	Find the area which is inside the circle $r=3a\cos\theta$ and outside the cardioids $r=a(1+\cos\theta)$.	**BTL -2**	Understanding

11.(b)	Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$	**BTL -5**	Evaluating
12.(a)	Find the area that lies inside the cardioids $r = a\,(1 + cos\theta)$ and outside the circle $r = a$ by double integral	**BTL -2**	Understanding
12.(b)	Find the volume of sphere $x^2 + y^2 + z^2 = r^2$using triple integral	**BTL -2**	Understanding
13.(a)	Evaluate $\iint_R \frac{xy}{\sqrt{x^2+y^2}} dxdy$ by converting into polar coordinates where R is the first quadrant part of the region bounded by two circles $x^2 + y^2 = a^2 and x^2 + y^2 = 4a^2$	**BTL -5**	Evaluating
13.(b)	Find the volume bounded by the cylinder $x^2 + y^2 = 1$ and the planes$x + y + z = 3\,, z = 0$	**BTL -2**	Understanding
14.(a)	Evaluate $\int_0^a \int_0^b \int_0^c (x + y + z) dxdydz$	**BTL -5**	Evaluating
14.(b)	Find the area enclosed by the curves $y^2 = 4ax$ and $x^2 = 4ay$	**BTL -2**	Understanding