VALLIAMMAI ENGINEERING COLLEGE

S.R.M. Nagar, Kattankulathur - 603203

DEPARTMENT OF MATHEMATICS

QUESTION BANK

II SEMESTER

(COMMON TO ALL BRANCHES)

MA6251- MATHEMATICS –II

Regulation – 2013

Academic Year – 2016 - 17

Prepared by

Dr.T.Isaiyarasi , Asst. Prof. (SLG)
Ms.C.V.Dhanya, Asst.Prof. (OG)
Ms.A.Karpagam, Asst.Prof. (OG)
Ms.G.Sasikala, Asst. Prof. (OG)

VALLIAMMAI ENGINEERING COLLEGE
SRM Nagar, Kattankulathur – 603 203.
DEPARTMENT OF MATHEMATICS

QUESTION BANK

SUBJECT : MA6251- MATHEMATICS-II

SEM / YEAR:II SEMESTER / I YEAR (COMMON TO ALL BRANCHES)

UNIT I- VECTORCALCULUS			
Gradient, divergence and curl – Directional derivative – Irrotational and Solenoidal vector fields – Vector integration – Green's theorem in a plane, Gauss divergence theorem and Stokes" theorem (excluding proofs) – Simple applications involving cubes and rectangular parallelepipeds.			
Q.No.	Question	BT Level	Competence
PART-A			
1	Find $\nabla\varphi$, if $\varphi = x^2 + y^2 + z^2$ at (1, -1, 1).	BTL-1	Remembering
2	Find the Directional derivative of $f = xyz$ at (1,1,1)in the direction $\vec{i} + \vec{j} + \vec{k}$.	BTL-1	Remembering
3	Find the Directional derivative of $\varphi = 4xz^2 + x^2yz$ at (1,-2,1) in the direction $2\vec{i} + 3\vec{j} + 4\vec{k}$.	BTL-1	Remembering
4	State Gauss Divergence theorem.	BTL-1	Remembering
5	State Stokes theorem.	BTL-1	Remembering
6	State Greens theorem	BTL-1	Remembering
7	Give the unit normal vector to the surface $xyz = 2$ at (2, 1, 1).	BTL-2	Understanding
8	Give the unit normal vector to the surface $x^2 + y^2 + z^2 = 1$ at (1,1,1).	BTL-2	Understanding
9	If $\varphi = 3xy - yz$., Give $grad\ \varphi$ at (1,1,1).	BTL-2	Understanding
10	If $\vec{r}$ is the position vector, Give $div\ \vec{r}$.	BTL-2	Understanding
11	Show that$\nabla(r^n) = nr^{n-2}\vec{r}$.	BTL-3	Applying
12	Show that the vector $\vec{F} = (y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$ is solenoidal .	BTL-3	Applying
13	Show that $curl(grad\ \varphi) = 0$.	BTL-3	Applying
14	If $\vec{F} = 3xy\vec{i} - y^2\vec{j}$, Evaluate $\int \vec{F}.d\vec{r}$, where C is the arc of the parabola $y = 2x^2$ from the point (0,0) to the point (1,2).	BTL-4	Analyzing
15	If $\vec{F} = (x^2)\vec{i} + (xy^2)\vec{j}$, evaluate $\int \vec{F}.d\vec{r}$ from (0,0) to (1,1) along the path $y = x$.	BTL-4	Analyzing
16	Using Green's theorem evaluate $\int_C [(2x^2 - y^2)dx + (x^2 + y^2)\,dy$ where C is the boundary of the square enclosed by the lines $x = 0, y = 0, x = 2, y = 3$.	BTL-4	Analyzing
17	Is the position vector $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ irrotational? Justify.	BTL-5	Evaluating
18	Evaluate using Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + +yz\vec{k}$ taken over the cube $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$.	BTL-5	Evaluating
19	What is the value of m if the vector $\vec{F} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x + mz)\vec{k}$ is solenoidal	BTL- 6	Creating
20	What is the value of a, b, c if the vector $\vec{F} = (x + y + az)\vec{i} + (by + 2y - z)\vec{j} + (-x + cy + 2z)\vec{k}$ may be irrotational.	BTL- 6	Creating
PART-B			
1(a)	Verify Gauss divergence theorem for $\vec{F} = x^2\vec{i} + y^2\vec{j} + z^2\vec{k}$ taken over the cube bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 1$ and $z = 1$	BTL-2	Understanding
1(b)	Find the Directional Derivative of $\varphi = x^2yz + 4xz^2 + xyz$ at (1, 2, 3) in the direction of $2\vec{i} + \vec{j} - \vec{k}$.	BTL-1	Remembering
2(a)	Verify Gauss divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube	BTL-4	Analyzing

	bounded by the planes $x = 0, y = 0, z = 0, x = 1, y = 1$ and $z = 1$.		
2(b)	**Find the Directional Derivative of $\varphi = x^2yz + 4xz^2$ at the point P(1,- 2, -1) in the direction of PQ where Q is (3,-3,-3).**	**BTL-5**	**Evaluating**
3(a)	**Verify divergence theorem for $\vec{F} = (x^3 - yz)\,\vec{i} - 2x^2y\,\vec{j} + 2\,\vec{k}$ over the cube formed by the planes $x = 0, y = 0, z = 0, x = a, y = a$ and $z = a$.**	**BTL-2**	**Understanding**
3(b)	**Find the maximum directional derivative of $\varphi = xyz^2$ at the point (1,0,3)**	**BTL-5**	**Evaluating**
4(a)	**Verify divergence theorem for $\vec{F} = (2x - z)\vec{i} + (x^2y)\vec{j} - xz^2\vec{k}$ over the cube formed by the planes $x = 0, y = 0, z = 0, x = 1, y = 1$ and $z = 1$.**	**BTL-4**	**Analyzing**
4(b)	**Find the unit normal vector to the surface $x^2y + 2xz^2 = 8$ at (1,0, 2).**	**BTL-5**	**Evaluating**
5(a)	**Verify divergence theorem for $\vec{F} = (2xz)\vec{i} + (2yz)\vec{j} + 2xz\,\vec{k}$ over the cube formed by the planes $x = 0, y = 0, z = 0, x = 1, y = 1$ and $z = 1$.**	**BTL-4**	**Analyzing**
5(b)	**Find the value of n such that the vector $r^n \vec{r}$ is both solenoidal and irrotational.**	**BTL-5**	**Evaluating**
6(a)	**Verify Green's theorem in the plane for $\int_c [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by x = 0, y = 0, x + y = 1.**	**BTL-4**	**Analyzing**
6(b)	**Show that the surfaces $5x^2 - 2yz - 9x = 0$ and $4x^2y + z^3 - 4 = 0$ are orthogonal at the point (1,-1, 2).**	**BTL-1**	**Remembering**
7(a)	**Verify Green's theorem and find $\int_C x^2(1 + y)dx + (y^3 + x^3)\,dy$ where C is the square bounded by $x = \pm a, y = \pm a$.**	**BTL-1**	**Remembering**
7(b)	**Find the values of a and b so that the surfaces $ax^3 - by^2z = (a+3)x^2$ and $4x^2y - z^3 = 11$ may cut orthogonally at (2,-1,-3).**	**BTL-2**	**Understanding**
8(a)	**Evaluate $\int_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where C is the square bounded by the lines $x = 0, x = 2, y = 0$ and $y = 3$ by Green's theorem.**	**BTL-4**	**Analyzing**
8(b)	**Find the angle between the normals to the surface $xy = z^2$ at the points (1,4,2) and (-3,-3,3).**	**BTL-1**	**Remembering**
9(a)	**Verify Green's theorem in the plane for $\int_c [(3x^2 - 8y^2)dx + (4y - 6xy)dy]$ where C is the boundary of the region bounded by $x = y^2, y = x^2$.**	**BTL-5**	**Evaluating**
9(b)	**Find the angle between the normals to the surface $x^2 - y^2 - z^2 = 11$ and $xy + yz - zx = 18$ at the points (6,4,3).**	**BTL-1**	**Remembering**
10(a)	**Find its scalar potential, if the vector field $\vec{F} = (x^2 + xy^2)\,\vec{i} + (y^2 + x^2y)\,\vec{j}$ is irrotational.**	**BTL-5**	**Evaluating**
10(b)	**Compute $\int_C (xy - x^2)dx + x^2y\,dy$ over the triangle bounded by the lines $y = 0, x = 1, y = x$ and verify Greens theorem.**	**BTL-1**	**Remembering**
11(a)	**Verify Stokes theorem for $\vec{F} = (x^2 - y^2)\,\vec{i} + 2xy\,\vec{j}$ in the rectangular region of $z = 0$ plane bounded by the lines $x = 0, y = 0, x = a$ and $y = b$.**	**BTL-2**	**Understanding**
11(b)	**Show that $\vec{F} = (2xy - z^2)\,\vec{i} + (x^2 + 2yz)\,\vec{j} + (y^2 - 2zx)\,\vec{k}$ is irrotational and hence find its scalar potential.**	**BTL-3**	**Applying**
12(a)	**Show that Stokes theorem is verified for $\vec{F} = (y - z)\vec{i} + yz\,\vec{j} - xz\,\vec{k}$ where S is the surface bounded by the planes $x = 0, x = 1, y = 0, y = 1, z = 0$ *and* $z = 1$ above the xy-plane.**	**BTL-3**	**Applying**
12(b)	**Show that $\vec{F} = (y^2 + 2xz^2)\,\vec{i} + (2xy - z)\,\vec{j} + (2x^2z - y + 2z)\,\vec{k}$ is irrotational and hence find and formulate its scalar potential.**	**BTL-4**	**Analyzing**

13(a)	Find $\int_C \vec{F}.d\vec{r}$ using Stoke's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ over the open surfaces of the cube $x = 0, y = 0, z = 0, x = 1, y = 1, z = 1$ not included in the XOY plane.	BTL-1	Remembering
13(b)	Find the work done in moving a particle in the vector field $\vec{F} = (y+3)\vec{i} + xz\vec{j} + (yz - x)\vec{k}$ along the curve $x = 2t^2, y = t, z = t^3$ from (0,0,0) to (2,1,1).	BTL-3	Applying
14(a)	Verify Stokes theorem for $\vec{F} = (x - y + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ where S is the surface bounded by the planes $x = 0, x = 2, y = 0, y = 2, z = 0$ *and* $z = 2$ above the xy-plane.	BTL-1	Remembering
14(b)	Prove that $\nabla^2(r^n) = n(n+1)r^{n-2}$	BTL-3	Applying

UNIT II ORDINARY DIFFERENTIAL EQUATIONS

Higher order linear differential equations with constant coefficients – Method of variation of parameters – Cauchy's and Legendre's linear equations – Simultaneous first order linear equations with constant coefficients

PART –A

1	Find the P.I of $(D-1)^2 y = \sinh 2x.$	BTL-1	Remembering
2	Find the P.I of $(D^2+1)y = \cos 2x.$	BTL-1	Remembering
3	Find the P.I of $(D^2 + 1)y = sinx.$	BTL-1	Remembering
4	Find the particular Integral for $(D^2+2D-1)y = x.$	BTL-1	Remembering
5	Find the P.I of $(D^2+2)y = x^2.$	BTL-1	Remembering
6	Find the P.I of $(D^2+4D+5)y = e^{-2x}$	BTL-1	Remembering
7	Estimate the P.I of $(D^2 + 5D + 4)y = sin2x.$	BTL-2	Understanding
8	Estimate the P.I of $(D^2 - 4D + 4)y = e^{2x}.$	BTL-2	Understanding
9	Estimate the P.I of $(D^3 + 3D^2 + 3D + 1)y = e^{-x}.$	BTL-2	Understanding
10	Find the complementary function of $(D^2 + 4)y = sin2x.$	BTL-2	Understanding
11	Solve $(D^4-1)y = 0.$	BTL-3	Applying
12	Solve $Dx = -wy$; $Dy = wx$	BTL-3	Applying
13	Solve Dx + y = e^t, $x - Dy = t.$	BTL-3	Applying
14	Find the complementary function of $y'' - 4y' + 4y = 0$	BTL-4	Analyzing
15	Solve $(D^2+a^2)y=0$	BTL-3	Applying
16	Convert $(x^2D^2 - 2xD + 4)y = 0$ in to differential equations with constant coefficients	BTL-6	Creating
17	Test whether the equation $x^2y'' + xy' = x$ is linear equation with constant coefficients if not convert.	BTL-5	Evaluating
18	Solve $(D^4 + D^3 + D^2)y = 0$	BTL-5	Evaluating
19	Rewrite the equation $(2x + 5)^2D^2 - 6(2x + 5)D + 8y = 6x$ into the linear equation with constant coefficients.	BTL-6	Creating
20	Rewrite the equation $(2x - 1)^2D^2 - 4(2x - 1)D + 8y = 8x$ into the linear equation with constant coefficients	BTL-6	Creating

Part-B

1(a)	Identify the solution of $(D^2 - 2D + 1)y = coshx.$	BTL-1	Remembering
1(b)	Using the method of variation of parameter to Evaluate $(D^2 + 1)\ y = x\,sinx.$	BTL-2	Understanding
2(a)	Identify the solution of $(D^2-4D+13)y = e^{2x}\sin 3x + (x^2+x+9).$	BTL-1	Remembering
2(b)	Using the method of variation of parameter to Evaluate $(D^2 + 25)$ y = sec5x.	BTL-2	Understanding

3(a)	Identify the solution of $(D^3-7D-6)y=(1+x)e^{2x}$	BTL-1	Remembering
3(b)	Solve $y''-2y'+y=e^x\log x$, Using the method of variation of parameters.	BTL-3	Applying
4(a)	Give the complimentary function and particular integral of $(D^2-3D+2)y=x\cos x$.	BTL-2	Understanding
4(b)	Using the method of variation of parameters find the solution of $(D^2+2D+1)=\frac{e^{-x}}{x^2}$	BTL-4	Analyzing
5(a)	Solve $(x^2D^2-xD+1)y=x\sin(\log x)+\frac{1}{x}$.	BTL-3	Applying
5(b)	Evaluate the simultaneous equations $\frac{dx}{dt}+2x-3y=5t, \frac{dy}{dt}-3x+2y=2e^{2t}$ given that $x(0)=0, y(0)=-1$.	BTL-5	Evaluating
6(a)	Give the general solution of $x^2\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=\sin(\log x^2)$.	BTL-1	Remembering
6(b)	Solve: $\frac{dx}{dt}+2y=\sin 2t, \frac{dy}{dt}-2x=\cos 2t$.	BTL-3	Applying
7(a)	Find the solution of $(2x+3)^2\frac{d^2y}{dx^2}-(2x+3)\frac{dy}{dx}-12y=6x$.	BTL-1	Remembering
7(b)	Formulate the ODE and hence solve $(x\ ^2D^2 + 4xD + 2)y = x^2 + \frac{1}{x^2}$	BTL-6	Creating
8(a)	Solve $y''+y= cosec\ x$ by method of variation of parameters	BTL-3	Applying
8(b)	Identify the solution of $D^2x- 5x + 3y = \sin t$, $D^2y +5y -3x = t$	BTL-1	Remembering
9(a)	Solve the differential equation $y''+y= \sec x$ by method of variation of parameters	BTL-3	Applying
9(b)	Evaluate the general solution of (x^2D^2-xD+1) y = $\left(\frac{logx}{x}\right)^2$	BTL-5	Evaluating
10(a)	Solve the differential equation $y''-2y'+2y=e^x\ tanx$ by method of variation of parameters	BTL-3	Applying
10(b)	Formulate the ODE and hence solve ($2x+5)2\ y''-6(2x + 5)\ y'+8y=6x$.	BTL-6	Creating
11(a)	Solve the equation $(D^2+4D+3)y=e^{-x}sinx$	BTL-3	Applying
11(b)	Identify the solution of $Dx - y = t, Dy + x = 1$	BTL-1	Remembering
12(a)	Solve $\frac{dx}{dt}+y=e^t, x-\frac{dy}{dt}= t$	BTL-3	Applying
12(b)	Solve $(x^2D^2+2xD)y=logx$	BTL-3	Applying
13(a)	Solve $(D^2-2D+1)y = xe^x sinx$	BTL-3	Applying
13(b)	Solve $((x+3)^2D^2-4(x+3)D+6)y=log(x+3)$	BTL-3	Applying
14(a)	Solve $\frac{dy}{dt}+x=e^{2t}, \frac{dx}{dt}+y=t$	BTL-3	Applying
14(b)	Solve $(x^2D^2-2xD-4)y=32(logx)^2$	BTL-3	Applying

UNIT III LAPLACE TRANSFORM

Laplace transform – Sufficient condition for existence – Transform of elementary functions – Basic properties – Transforms of derivatives and integrals of functions - Derivatives and integrals of transforms - Transforms of unit step function and impulse functions – Transform of periodic functions. Inverse Laplace transform -Statement of Convolution theorem – Initial and final value theorems – Solution of linear ODE of second order with constant coefficients using Laplace transformation techniques.

PART-A

1	State the sufficient conditions for the existence of Laplace transform.	BTL-1	Remembering
2	State first and second shifting theorem.	BTL-1	Remembering
3	State and prove change of scale property	BTL-1	Remembering
4	State Initial value and final value theorems.	BTL-1	Remembering
5	State Convolution theorem	BTL-1	Remembering
6	Tell whether $L\left[\frac{\cos t}{t}\right]$ exist? Justify.	BTL-1	Remembering

7	**Find the inverse Laplace transform of F(s) = $\frac{1}{s(s-2)}$**	**BTL-2**	**Understanding**
8	**Estimate L[t cost]**	**BTL-2**	**Understanding**
9	**Estimate** $L\left[\frac{\sin at}{t}\right]$	**BTL-2**	**Understanding**
10	**Find** $L^{-1}\left[\cot^{-1} s\right]$	**BTL-2**	**Understanding**
11	**Apply and verify the initial value theorem for the function f(t) = 3 e^{-2t}**	**BTL-3**	**Applying**
12	**Apply and verify the final value theorem of the function f(t) = t^2 e^{-3t}**	**BTL-3**	**Applying**
13	**Give example of two functions for which Laplace Transform do not exist?**	**BTL-3**	**Applying**
14	**Verify initial value theorem for the function 1+e^{-2t}.**	**BTL-4**	**Analyzing**
15	**Find** $L^{-1}\left[\frac{e^{at}-e^{-bt}}{t}\right]$	**BTL-4**	**Analyzing**
16	**Find** $L^{-1}\left[\log\frac{s+1}{s-1}\right]$	**BTL-4**	**Analyzing**
17	**Evaluate** $L^{-1}\left[\frac{1}{(s+2)^4}\right]$	**BTL-5**	**Evaluating**
18	**Evaluate** $L^{-1}\left[\frac{3s}{2s+9}\right]$	**BTL-5**	**Evaluating**
19	**Formulate** $L^{-1}\left[\frac{3s+2}{s^2-4}\right]$	**BTL- 6**	**Creating**
20	**Formulate** $L^{-1}\left[\frac{1}{s(s-4)}\right]$	**BTL- 6**	**Creating**
	PART-B		
1(a)	**Estimate L[f(t)], if f(t)=** $\begin{cases}\sin\omega t, & for\ 0<t<\frac{\pi}{\omega}\\ 0, & for\frac{\pi}{\omega}<t<\frac{2\pi}{\omega}\end{cases}$ **and** $f\left(t+\frac{2\pi}{\omega}\right)=f(t)$**, for all t.**	**BTL-2**	**Understanding**
1(b)	**Identify the Inverse Laplace transform of** $\left[\tan^{-1}\left(\frac{2}{s}\right)+\cot^{-1}\left(\frac{s}{3}\right)\right]$	**BTL-1**	**Remembering**
2(a)	**Give L [f(t)], if f(t)=** $\begin{cases}t, & for\ 0\le t\le c\\ 2c-t, & for\ c<t<2c\end{cases}$ **and** $f(t+2c)=f(t)$**), for all t.**	**BTL-2**	**Understanding**
2(b)	**Find the inverse Laplace Transform of log**$\left(\frac{s^2+a^2}{s^2+b^2}\right)$	**BTL-5**	**Evaluating**
3(a)	**Identify the Laplace transform of the square- wave function of period a**	**BTL-1**	**Remembering**

	defined as f(t)= $\begin{cases} 1, when\ 0 < t < a/2 \\ -1, when\ a/2 < t < a \end{cases}$		
3(b)	**Find** $L^{-1}\left[\frac{5s+3}{(s^2+2s+5)(s-1)}\right]$	**BTL-4**	**Analyzing**
4(a)	**Find the Laplace transform of the square- wave function of period a defined as f(t)=** $\begin{cases} K, when\ 0 < t < a \\ -K, when\ a < t < 2a \end{cases}$ **and** $f(t+2a) = f(t)$**, for all t.**	**BTL-5**	**Evaluating**
4(b)	**Find f(t), if** $L(f(t)) = \frac{s}{(s+2)^2}$	**BTL-5**	**Evaluating**
5(a)	**Identify the Laplace Transform of the function** $[t\ Sin3t\ Cos\ 2t]$	**BTL-1**	**Remembering**
5(b)	**Give L [f(t)], if f(t)=** $\begin{cases} t,\ for\ 0 \le t \le 1 \\ 2-t,\ for 1 < t < 2 \end{cases}$ **and** $f(t+2) = f(t)$**, for all t.**	**BTL-5**	**Evaluating**
6(a)	**Find the Laplace transform of f(t) if** $f(t) = e^t, 0 < t < 2\pi$ **and** $f(t+2\pi) = f(t)$	**BTL-4**	**Analyzing**
6(b)	**Identify the Laplace Transform of the function** $\left[\frac{1-\cos t}{t}\right]$	**BTL-1**	**Remembering**
7(a)	**Apply initial and final value theorem for the verification of the function f(t) = 1+** e^{-t} **(sint + cost).**	**BTL-3**	**Applying**
7(b)	**Using Convolution theorem, Evaluate** $L^{-1}\left[\frac{1}{s(s^2+1)}\right]$	**BTL-5**	**Evaluating**
8(a)	**Using convolution theorem, find** $L^{-1}\left[\frac{4}{(s^2+2s+5)^2}\right]$	**BTL-4**	**Analyzing**
8(b)	**Give the general solution of (**D^2 **+ 9) y = cos2t, given that y (0) =1, y** $\left(\frac{\pi}{2}\right)$ **=-1.**	**BTL-2**	**Understanding**
9(a)	**Using Convolution theorem calculate the inverse Laplace transform of** $L^{-1}\left[\frac{s^2}{(s^2+1)(s^2+4)}\right]$	**BTL- 6**	**Creating**
9(b)	**Give the general solution of (**D^2 **+ 4D + 4) y =** e^{-t}**, given that y (0) =0,y' (0) =0.**	**BTL-2**	**Understanding**
10(a)	**Solve y''-3y'+2y= 4t +** e^{3t} **when y' (0) = -1and y (0) = 1 using Laplace transforms.**	**BTL-5**	**Evaluating**
10(b)	**Apply convolution theorem, find** $L^{-1}\left[\frac{s}{(s^2+a^2)^2}\right]$	**BTL-3**	**Applying**
11(a)	**Formulate and solve using Laplace transforms, (**D^2**+D) y =** t^2**+2t, given that y=4, y'= -2 when t=0**	**BTL-5**	**Evaluating**
11(b)	**Using Convolution theorem calculate the inverse Laplace transform of**	**BTL- 6**	**Creating**

	$L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$		
12(a)	Using Convolution theorem calculate the inverse Laplace transform of $L^{-1}\left[\frac{1}{(s+1)(s^2+1)}\right]$	BTL- 6	Creating
12(b)	using Laplace transforms, solve y''-3y'-4y= $2e^{-t}$ when y' (0) = 1and y(0)= 1.	BTL-5	Evaluating
13(a)	Using Convolution theorem calculate the inverse Laplace transform of $L^{-1}\left[\frac{2}{(s+1)(s^2+4)}\right]$	BTL- 6	Creating
13(b)	Using Laplace transforms, solve y''-2y'+y= e^t when y' (0) = -1and y(0)=2	BTL-4	Analyzing
14(a)	Identify the Inverse Laplace transform of $\left[\frac{3s^2+16s+26}{s(s^2+4s+13)}\right]$	BTL-1	Remembering
14(b)	Evaluate $L\left[\frac{\cos at-\cos bt}{t}\right]$	BTL-5	Evaluating

UNIT IV ANALYTIC FUNCTIONS

Functions of a complex variable – Analytic functions: Necessary conditions – Cauchy-Riemann equations and sufficient conditions (excluding proofs) – Harmonic and orthogonal properties of analytic function – Harmonic conjugate – Construction of analytic functions – Conformal mapping: $w = z+k, kz, 1/z, z^2, e^z$ and bilinear transformation.

PART-A

1	Examine if f(z)= z^3 analytic ?	BTL-1	Remembering
2	Identify the constants a,b,c if $f(z) = x + ay + i(bx + cy)$ is analytic.	BTL-1	Remembering
3	Define conformal mapping.	BTL-1	Remembering
4	Can $u = 3x^2y - y^3$ be the real part of an analytic function? Justify your answer	BTL-1	Remembering
5	State necessary and sufficient condition for Cartesian coordinates in Cauchy-Riemann Equation	BTL-1	Remembering
6	Identify the invariant point of the bilinear transformation $w = \frac{2z+6}{z+7}$	BTL-1	Remembering
7	Estimate the invariant points of the transformation $w=\frac{z-1}{z+1}$	BTL-2	Understanding
8	Estimate the invariant point of the bilinear transformation $w = \frac{1+z}{1-z}$	BTL-2	Understanding
9	Give the image of the circle $\|z\| = 3$ under the transformation w = 5z.	BTL-2	Understanding
10	Under the transformation $w = \frac{1}{z}$ give the image of the circle $\|z-1\| = 1$ in the complex plane.	BTL-2	Understanding
11	Show that $\|z\|^2$ is not analytic at any point.	BTL-3	Applying
12	Show that an analytic function in a region R with constant imaginary part is constant.	BTL-3	Applying

13	Show that $u = 2x - x^3 + 3xy^2$ is harmonic and determine its harmonic conjugate.	BTL-3	Applying
14	If f(z) is an analytic function whose real part is constant, Point out f(z) is a constant function.	BTL-4	Analyzing
15	Explain that a bilinear transformation has at most 2 fixed points.	BTL-4	Analyzing
16	Examine whether the function xy^2 can be real part of analytic function.	BTL-4	Analyzing
17	Test the analyticity of the function $f(z) = e^{-z}$	BTL-5	Evaluating
18	Evaluate the image of hyperbola $x^2 - y^2 = 1$ under the transformation $w = \frac{1}{z}$	BTL-5	Evaluating
19	Formulate the critical points of the transformation $w = z + \frac{1}{z}$	BTL-6	Creating
20	Formulate the bilinear transformation which maps $z = 0, -i, -1$ into $w = i, 1, 0$ respectively.	BTL-6	Creating
	PART-B		
1(a)	Given that $= \frac{sin2x}{cosh2y - cos2x}$, Estimate the analytic function.	BTL-2	Understanding
1(b)	Find the image of $\|z\| = 2$ under the transformation $(i) w = z + 3 + 2i \; (ii)\; w = 3z$	BTL-4	Analysing
2(a)	Estimate the analytic function $w = u + iv$ if $u = e^x(xcos2y - ysin2y)$.	BTL-2	Understanding
2(b)	Formulate the image of $\|z + 1\| = 1$ under the map $w = 1/z$.	BTL-6	Creating
3(a)	Estimate the analytic function $f(z) = u + iv$ given the imaginary part is $v = x^2 - y^2 + \frac{x}{x^2+y^2}$.	BTL-2	Understanding
3(b)	Prove that an analytic function with constant modulus is constant.	BTL-1	Remembering
4(a)	Show that the function $u(x, y) = 3x^2y + 2x^2 - y^3 - 2y^2$ is harmonic. Find also the conjugate harmonic function v.	BTL-3	Applying
4(b)	Show that the transformation $w = \frac{1}{z}$ maps, in general, circles and straight lines into circles and straight lines. Point out the circles and straight lines are transformed into straight lines and circles respectively.	BTL-1	Remembering
5(a)	Estimate the analytic function $w = u + iv$ if $u - v = e^x(cosy - siny)$	BTL-2	Understanding
5(b)	Under the transformation $w = \frac{1}{z}$, find the image of a region (i) $x > c$ where $c > 0$, (ii) $y > c$, where $c < 0$	BTL-4	Analysing
6(a)	Solve the bilinear transformation that maps the point $z_1 = i$., $z_2 = -1$, $z_3 = 1$into the points $w_1 = 0, w_2 = 1, w_3 = \infty$ respectively.	BTL-5	Evaluating
6(b)	Identify the image of the infinite strip $(i) 0 \le y \le \frac{1}{2}$ $(ii) \frac{1}{4} < y < \frac{1}{2}$ under the transformation $= 1/z$.	BTL-2	Understanding
7(a)	Determine the analytic function $w = u + iv$ given that $3u + 2v = y^2 - x^2 + 16xy$.	BTL-4	Analysing
7(b)	Formulate the image of $\|z - 2\| = 2$ under the map $w = z^2$	BTL-6	Creating
8(a)	Point out the bilinear transformation that maps the point $z_1 = 1$, $z_2 = i, z_3 = -1$ into the points $w_1 = i, w_2 = 0, w_3 = -i$ respectively.	BTL-3	Applying
8(b)	Show that the image of $\|z - 1\| = 1$ under the transformation $w = z^2$ is	BTL-1	Remembering

	the cardioid $R = 2(1 + cos\varphi)$		
9(a)	Give the bilinear transformation which maps $z = 1, 0, -1$ into $w = 0, -1, \infty$ respectively. What are the invariant points of the transformation?	BTL-2	Understanding
9(b)	If $u = x^2 - y^2, v = -\frac{y}{x^2+y^2}$, prove that u and v are harmonic functions but u+iv is not an analytic function.	BTL-5	Ebvaluating
10(a)	Identify the bilinear transformation that maps $1 + i, -i, 2 - i$ at the z-plane into the points $0, 1, i$ of the w-plane.	BTL-1	Remembering
10(b)	Determine the analytic function $f(z) = u + iv$ such that $2u + v = e^x(cosy - siny)$		
11(a)	Identify the bilinear mapping which maps $-1, 0, 1$ of the z-plane onto $-1, i, 1$ of the w-plane. Show that under this mapping the upper half of z-plane maps onto the interior of unit circle $\|w\| = 1$.	BTL-1	Remembering
11(b)	If $w = f(z)$ is analytic then Show that $\left(\frac{\partial^2}{\partial x^2}\right) + \left(\frac{\partial^2}{\partial y^2}\right) \log\|f(z)\| = 0$.	BTL-6	Creating
12(a)	Identify the bilinear mapping which maps $1, i, -1$ of the z-plane onto $0, 1, \infty$ of the w-plane . Show that the transformation maps the interior of the unit circle of the z-plane onto the upper half of the w-plane.	BTL-1	Remembering
12(b)	If f (z) is a regular function of z, Show that $\nabla^2\|f(z)\|^2 = 4\|f'(z)\|^2$.	BTL-3	Applying
13(a)	If $w = u(x, y) + iv(x, y)$ is an analytic function , show that the curves of the family $u(x, y) = a$ and the curves of the family $v(x, y) = b$, cut orthogonally where a and b are varying constants.	BTL-2	Understanding
13(b)	Solve the bilinear transformation that maps the point $z_1 = 0, z_2 = 1, z_3 = \infty$ into the points $w_1 = i, w_2 = 1, w_3 = -i$ respectively.	BTL-5	Analyzing
14(a)	If f (z) = u + iv is an analytic function of z, then formulate that $\nabla^2[\log\|f'(z)\|] = 0$.	BTL-6	Creating
14(b)	Solve the bilinear transformation that maps the point $z_1 = 0, z_2 = 1, z_3 = \infty$ into the points $w_1 = -5, w_2 = -1, w_3 = 3$ respectively. What are invariant points of transformation.	BTL-5	Analyzing

UNIT V COMPLEX INTEGRATION

Complex integration – Statement and applications of Cauchy's integral theorem and Cauchy's integral formula – Taylor's and Laurent's series expansions – Singular points – Residues – Cauchy's residue theorem – Evaluation of real definite integrals as contour integrals around unit circle and semi-circle (excluding poles on the real axis).

PART –A

1	State Cauchy's integral theorem	BTL-1	Remembering
2	Identify the type of singularity of function $Sin\left(\frac{1}{1-z}\right)$.	BTL-1	Remembering
3	State Cauchy's residue theorem and Cauchy's integral formula	BTL-1	Remembering
4	Identify the value of $\int_C e^z dz$, where C is |z| = 1?	BTL-1	Remembering
5	Estimate the residue of the function $f(z) = \frac{4}{z^3(z-2)}$ at a simple pole	BTL-2	Understanding
6	Give the Laurent's series of $f(z) = \frac{1}{z(1-z)}$ valid in the region $\|z+1\| < 1$.	BTL-2	Understanding
7	Give the Laurent's series expansion of $f(z) = \frac{e^z}{(z-1)^2}$ about $z = 1$.	BTL-2	Understanding
8	Give the Taylor's series for $f(z) = Sin\, z$ about $z = \frac{\pi}{4}$.	BTL-2	Understanding

9	**Calculate the residue at $z = 0$ of** $f(z) = \frac{1-e^{z}}{z^{3}}$	**BTL-3**	**Applying**
10	**Calculate the residue of the function** $\frac{z-3}{(z+1)(z+2)}$ **at poles.**	**BTL-3**	**Applying**
11	**Determine the residues at poles of the function** $f(z) = \frac{z+4}{(z-1)(z-2)}$.	**BTL-3**	**Applying**
12	**Expand** $\frac{1}{z(z-1)}$ **as Laurent's series about z = 0 in the annulus 0 < |z| < 1.**	**BTL-4**	**Analyzing**
13	**Obtain the expansion of** $\log(1+z)\ when \|z\| < 1.$	**BTL-4**	**Analyzing**
14	**Evaluate** $\int_C \frac{z}{(z-2)} dz$ **where C is a)** $\|z\| = 1$ **b)** $\|z\| = 3$	**BTL-4**	**Analyzing**
15	**Evaluate** $\oint_C \frac{z+2}{z} dz\ \ where\ C\ is\ the\ circle\ \|z\| = 2$ **in the z –plane.**	**BTL-5**	**Evaluating**
16	**Evaluate** $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3} dz\ where\ C\ is\ \|z\| = 4$ **using Cauchy's integral formula.**	**BTL-5**	**Evaluating**
17	**Integrate** $\int_C \frac{dz}{z+4}$ **where C is the circle | Z | =2.**	**BTL-6**	**Creating**
18	**Integrate** $\int_C \frac{e^z}{z-1} dz\ if\ C\ is\ \|z\| = 2.$	**BTL- 6**	**Creating**
19	**Expand** $f(z) = \frac{1}{z^2}$ **as Taylor's series about the point** $z = 2$	**BTL-4**	**Analyzing**
20	**Evaluate the residues of** $f(z) = tanz$ **at its isolated singularities.**	**BTL-5**	**Evaluating**
	PART -B		
1(a)	**Find the Taylor's series to represent the function** $\frac{z^2-1}{(z+2)(z+3)} in \|z\| < 2.$	**BTL-4**	**Analyzing**
1(b)	**Using contour integration estimate** $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$ **a> 0, b > 0.**	**BTL-2**	**Understanding**
2(a)	**Identify the Taylor's series to represent the function** $\frac{1}{(z+2)(z+3)} in \|z\| < 2.$	**BTL-1**	**Remembering**
2(b)	**Using Contour Integration evaluate** $\int_{-\infty}^{\infty} \frac{x^2 dx}{(x^2+4)(x^2+9)}$	**BTL-2**	**Understanding**
3(a)	**Identify the Laurent's series expansion of** $f(z) = \frac{z^2-1}{z^2+5z+6}$ **in the region** $\|z\| < 2\|$ **and** $2 < \|z\| < 3$	**BTL-1**	**Remembering**
3(b)	**Apply the calculus of residues to prove that** $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{4a^3}$	**BTL 3**	**Applying**
4(a)	**Identify the Laurent's series expansion for the function** $f(z) = \frac{4z}{(z^2-1)(z-4)}$ **in the regions** $2 < \|z-1\| < 3\ and\ \|z-1\| > 4$	**BTL-1**	**Remembering**
4(b)	**Apply the calculus of residues to prove that** $\int_0^{\infty} \frac{dx}{(x^4+a^4)} = \frac{\pi\sqrt{2}}{4a^3}$	**BTL 3**	**Applying**
5(a)	**Using Laurent's series, find** $\frac{1}{z(z-1)}$ **valid in (i)** $\|z+1\| < 1$ **(ii)** $1 < \|z+1\| < 2$ **(iii)** $\|z+1\| > 1$	**BTL-2**	**Understanding**
5(b)	**Evaluate using contour integration** $\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$	**BTL-5**	**Evaluating**
6(a)	**Identify the Laurent's series of** $f(z) = \frac{(z+3)}{(z-1)(z-4)}$ **, valid in** $\|z\| > 2$**and** $0 < \|z-1\| < 1$	**BTL-1**	**Remembering**
6(b)	**Apply the calculus of residues to evaluate** $\int_0^{\infty} \frac{x sinmx}{x^2+a^2} dx, a > 0, m > 0$	**BTL 3**	**Applying**

7(a)	Identify the Laurent's series expansion for the function $f(z)=\dfrac{7z-2}{(z+1)z(z-2)}$ in the region 1 < |z + 1| < 3.	BTL-1	Remembering
7(b)	Apply the calculus of residues to evaluate $\int_0^\infty \frac{xsinx}{(x^2+1)(x^2+4)}dx$	BTL 3	Applying
8(a)	Evaluate using contour integration $\int_0^\infty \frac{cosax}{(x^2+b^2)^2}dx, a>0, b>0$	BTL-5	Evaluating
8(b)	Expand as Laurent's series of the function $\frac{z}{(z^2-3z+2)}$ in the regions (i) $\|z\|<1$ (ii)$1<\|z\|<2$ (iii) $\|z\|>3$	BTL-4	Analyzing
9(a)	Evaluate using contour integration $\int_0^\infty \frac{cosx}{(x^2+a^2)(x^2+b^2)}dx, a>b>0$	BTL-5	Evaluating
9(b)	Applying Cauchy's integral formula solve $\int_C \frac{\cos\pi z^2+\sin\pi z^2}{(z-1)(z-2)}dz$, C is the circle |z | = 3.	BTL-3	Applying
10(a)	Evaluate $\int_0^{2\pi}\frac{d\theta}{(a+bsin\theta)}(a>0, b>0)$, using contour integration	BTL-5	Evaluating
10(b)	Using Cauchy's integral formula calculate $\int_C \frac{z+4}{z^2+2z+5}dz$ where C is the circle |z + 1 + i| = 2	BTL-3	Applying
11(a)	Formulate $\int_0^{2\pi}\frac{d\theta}{13+5\sin\theta}$, using the method of contour integration.	BTL-6	Creating
11(b)	Evaluate using Cauchy's integral formula $\int_C \frac{(z+1)}{(z-3)(z-1)}dz$ where C is the circle $\|z\|=2$	BTL-5	Evaluating
12(a)	Evaluate $\int_0^{2\pi}\frac{d\theta}{(a+bcos\theta)}(a>0, b>0)$, using contour integration	BTL-5	Evaluating
12(b)	Evaluate $\int_C \frac{z^2dz}{(z-1)^2(z+2)}$ $where\, C\, is\, \|z\|=3.$	BTL-5	Evaluating
13(a)	Evaluate $\int_0^{2\pi}\frac{cos3\theta d\theta}{(5+4cos\theta)}$ using contour integration	BTL-5	Evaluating
13(b)	If f(a) = $\int_C \frac{3z^2+7z+1}{(z-a)}dz$ where C is the circle |z | = 2, Identify f(3), f(1), $f'(1-i)$, $f''(1-i)$.	BTL-1	Remembering
14(a)	Evaluate $\int_0^{2\pi}\frac{d\theta}{(13+12cos\theta)}(a>0, b>0)$, using contour integration	BTL-5	Evaluating
14(b)	Evaluate $\int_C \frac{zdz}{(z-)(z-2)^2}$ where C is the circle $\|z-2\|=\frac{1}{2}$	BTL-4	Analyzing